B. PHARMACY 1ST SEMESTER

REMEDIAL MATHEMATICS 📊

UNIT – 4 ANALYTICAL GEOMETRY & INTEGRATION 📐

POINTS TO BE COVERED IN THIS TOPIC

- ➤ INTRODUCTION TO ANALYTICAL GEOMETRY 📍
- ➤ SIGNS OF THE COORDINATES ✏️
- ➤ DISTANCE FORMULA 📏
- ➤ STRAIGHT LINE 📈
- ➤ INTEGRATION ∫

📍 INTRODUCTION TO ANALYTICAL GEOMETRY

Analytical Geometry, also known as **Coordinate Geometry**, is a branch of mathematics that uses algebraic methods to study geometric properties and relationships. It bridges the gap between algebra and geometry by representing geometric figures using coordinates and equations.

The fundamental concept of analytical geometry is the **coordinate system**, which allows us to:

- Represent points in a plane using ordered pairs (x, y)
- Express geometric relationships through algebraic equations
- Solve geometric problems using algebraic methods

- Visualize algebraic equations as geometric curves

This branch of mathematics was developed by **René Descartes** and **Pierre de Fermat** in the 17th century, revolutionizing the way we approach geometric problems.

✏️ SIGNS OF THE COORDINATES

The **Cartesian coordinate system** divides the plane into four distinct regions called **quadrants**. The signs of coordinates vary depending on which quadrant a point lies in.

THE FOUR QUADRANTS

Quadrant	Position	x-coordinate	y-coordinate	Sign Convention
I	Upper Right	Positive (+)	Positive (+)	(+, +)
II	Upper Left	Negative (-)	Positive (+)	(-, +)
III	Lower Left	Negative (-)	Negative (-)	(-, -)
IV	Lower Right	Positive (+)	Negative (-)	(+, -)

SPECIAL POSITIONS

On the Axes:

- Points on the **positive x-axis** have coordinates (a, 0) where $a > 0$
- Points on the **negative x-axis** have coordinates (-a, 0) where $a > 0$
- Points on the **positive y-axis** have coordinates (0, b) where $b > 0$
- Points on the **negative y-axis** have coordinates (0, -b) where $b > 0$

- The **origin** has coordinates (0, 0)

Understanding Coordinate Signs: The sign of coordinates helps in:

- Determining the quadrant of a point
- Calculating distances correctly
- Understanding the direction of movement
- Solving geometric problems involving position

DISTANCE FORMULA

The **Distance Formula** is fundamental in analytical geometry for calculating the distance between two points in a coordinate plane.

DERIVATION AND FORMULA

For two points $A(x_1, y_1)$ and $B(x_2, y_2)$, the distance AB is given by:

Distance = $\sqrt{[(x_2 - x_1)^2 + (y_2 - y_1)^2]}$

DERIVATION PROCESS

The distance formula is derived using the **Pythagorean theorem**:

- Consider two points $A(x_1, y_1)$ and $B(x_2, y_2)$
- Draw perpendiculars to form a right triangle
- The horizontal distance = $|x_2 - x_1|$
- The vertical distance = $|y_2 - y_1|$
- Apply Pythagorean theorem: $(\text{Distance})^2 = (\text{horizontal distance})^2 + (\text{vertical distance})^2$

PROPERTIES OF DISTANCE

Key Properties:

- Distance is always **non-negative**
- Distance from A to B equals distance from B to A
- Distance is zero only when both points are identical
- The formula works in all quadrants

Applications:

- Finding perimeter of geometric figures
- Determining if points form specific shapes
- Calculating lengths of line segments
- Solving optimization problems

SPECIAL CASES

Distance from Origin: For a point P(x, y), distance from origin O(0, 0) = $\sqrt{(x^2 + y^2)}$

Distance Along Axes:

- If points lie on same vertical line: Distance = $|y_2 - y_1|$
- If points lie on same horizontal line: Distance = $|x_2 - x_1|$

📈 STRAIGHT LINE

A **straight line** is the shortest distance between two points and can be represented algebraically in various forms in analytical geometry.

🔺 SLOPE OR GRADIENT OF A STRAIGHT LINE

Definition: The **slope** (or gradient) of a straight line is a measure of its steepness and direction. It represents the ratio of vertical change to horizontal change between any two points on the line.

Formula: For a line passing through points (x_1, y_1) and (x_2, y_2): **Slope (m) = $(y_2 - y_1)/(x_2 - x_1)$**

CHARACTERISTICS OF SLOPE

Positive Slope: Line rises from left to right **Negative Slope:** Line falls from left to right
Zero Slope: Horizontal line **Undefined Slope:** Vertical line (denominator = 0)

GEOMETRIC INTERPRETATION

The slope represents:

- The **angle of inclination** with the x-axis
- The **rate of change** of y with respect to x
- The **tangent** of the angle made with positive x-axis

⚖️ CONDITIONS FOR PARALLELISM AND PERPENDICULARITY

Parallel Lines: Two lines are **parallel** if and only if they have **equal slopes**.

- If line 1 has slope m_1 and line 2 has slope m_2
- Lines are parallel when: $\mathbf{m_1 = m_2}$
- Parallel lines never intersect
- They maintain constant distance between them

Perpendicular Lines: Two lines are **perpendicular** if and only if the **product of their slopes equals -1**.

- If line 1 has slope m_1 and line 2 has slope m_2
- Lines are perpendicular when: $\mathbf{m_1 \times m_2 = -1}$
- This means: $\mathbf{m_2 = -1/m_1}$
- Perpendicular lines intersect at right angles (90°)

SPECIAL CASES FOR PERPENDICULARITY

- A **horizontal line** (slope = 0) is perpendicular to a **vertical line** (undefined slope)
- If one line is vertical, the other must be horizontal for perpendicularity

📊 SLOPE OF A LINE JOINING TWO POINTS

General Method: Given two points $A(x_1, y_1)$ and $B(x_2, y_2)$, the slope of line AB is: $\mathbf{m = (y_2 - y_1)/(x_2 - x_1)}$

Step-by-Step Process:

1. Identify coordinates of both points
2. Calculate difference in y-coordinates: $\Delta y = y_2 - y_1$
3. Calculate difference in x-coordinates: $\Delta x = x_2 - x_1$

4. Divide Δy by Δx to get slope
5. Ensure $x_2 \neq x_1$ to avoid division by zero

PRACTICAL APPLICATIONS

- **Engineering:** Calculating gradients of roads and ramps
- **Physics:** Determining rates of change
- **Economics:** Finding marginal costs and revenues
- **Architecture:** Designing roofs and structural elements

📋 SLOPE-INTERCEPT FORM OF A STRAIGHT LINE

Definition: The **slope-intercept form** is one of the most useful representations of a straight line equation.

Standard Form: $y = mx + c$

Where:

- **m** = slope of the line
- **c** = y-intercept (value of y when x = 0)
- **x, y** = coordinates of any point on the line

COMPONENTS EXPLANATION

Slope (m):

- Determines the steepness and direction
- Positive m: line slopes upward
- Negative m: line slopes downward

- Larger |m|: steeper line

Y-intercept (c):

- Point where line crosses y-axis
- Coordinates of y-intercept: (0, c)
- Represents initial value when x = 0

ADVANTAGES OF SLOPE-INTERCEPT FORM

- **Easy identification** of slope and y-intercept
- **Simple graphing** process
- **Quick comparison** between different lines
- **Convenient** for solving systems of equations

OTHER FORMS OF STRAIGHT LINE EQUATIONS

Form	Equation	When to Use
Point-Slope	$y - y_1 = m(x - x_1)$	When slope and one point are known
Two-Point	$(y - y_1)/(y_2 - y_1) = (x - x_1)/(x_2 - x_1)$	When two points are known
Standard	$Ax + By + C = 0$	General form for all lines

∫ INTEGRATION

Integration is one of the fundamental operations in calculus, representing the reverse process of differentiation.

🎯 INTRODUCTION TO INTEGRATION

Integration is a mathematical process that:

- Finds the **antiderivative** of a function
- Calculates the **area under curves**
- Solves **accumulation problems**
- Represents the **inverse operation** of differentiation

Historical Context: Integration was developed independently by **Newton** and **Leibniz** in the 17th century as part of calculus. It has become essential in:

- Physics (calculating work, energy, momentum)
- Engineering (finding centers of mass, moments)
- Economics (total cost, consumer surplus)
- Biology (population growth models)

📖 DEFINITION OF INTEGRATION

Indefinite Integration: If $F'(x) = f(x)$, then: $\int f(x)\, dx = F(x) + C$

Where:

- $\int$ = integration symbol
- **f(x)** = integrand (function to be integrated)
- **dx** = differential element
- **F(x)** = antiderivative
- **C** = constant of integration

Definite Integration: ∫[from a to b] f(x) dx = F(b) - F(a)

Where **a** and **b** are the limits of integration.

⚡ STANDARD FORMULAE FOR INTEGRATION

Function f(x)	∫ f(x) dx
k (constant)	**kx + C**
x^n ($n \neq -1$)	**x^(n+1)/(n+1) + C**
1/x	**ln
e^x	**e^x + C**
a^x	**a^x/ln(a) + C**
sin(x)	**-cos(x) + C**
cos(x)	**sin(x) + C**
$\sec^2(x)$	**tan(x) + C**
$\text{cosec}^2(x)$	**-cot(x) + C**
$1/\sqrt{(1-x^2)}$	**$\sin^{-1}(x) + C$**
$1/(1+x^2)$	**$\tan^{-1}(x) + C$**

📏 RULES OF INTEGRATION

1. Constant Multiple Rule: ∫ k·f(x) dx = k·∫ f(x) dx

2. Sum Rule: ∫ [f(x) + g(x)] dx = ∫ f(x) dx + ∫ g(x) dx

3. Difference Rule: ∫ [f(x) - g(x)] dx = ∫ f(x) dx - ∫ g(x) dx

4. Power Rule: ∫ x^n dx = x^(n+1)/(n+1) + C (for $n \neq -1$)

🔄 METHOD OF SUBSTITUTION

Purpose: To simplify complex integrals by changing variables.

Process:

1. **Identify substitution:** Choose u = g(x)
2. **Find differential:** du = g'(x) dx
3. **Substitute:** Replace all x terms with u terms
4. **Integrate:** Solve the simpler integral
5. **Back-substitute:** Replace u with original variable

When to Use Substitution:

- Composite functions present
- Chain rule pattern visible
- Inside function and its derivative both present

🧮 METHOD OF PARTIAL FRACTIONS

Purpose: To integrate rational functions by decomposing them into simpler fractions.

Process:

1. **Ensure proper fraction:** Degree of numerator < degree of denominator
2. **Factor denominator:** Find all factors
3. **Set up partial fractions:** Based on factor types
4. **Solve for constants:** Using algebraic methods
5. **Integrate:** Each partial fraction separately

Types of Factors:

- **Linear factors:** $(ax + b)$
- **Repeated linear factors:** $(ax + b)^n$
- **Quadratic factors:** $(ax^2 + bx + c)$
- **Repeated quadratic factors:** $(ax^2 + bx + c)^n$

⚙️ INTEGRATION BY PARTS

Formula: $\int u\,dv = uv - \int v\,du$

Selection Strategy (LIATE Rule): Choose **u** in order of priority:

1. **L**ogarithmic functions
2. **I**nverse trigonometric functions
3. **A**lgebraic functions
4. **T**rigonometric functions
5. **E**xponential functions

Process:

1. **Identify u and dv** using LIATE rule
2. **Calculate du** by differentiating u
3. **Calculate v** by integrating dv
4. **Apply formula:** $\int u\,dv = uv - \int v\,du$
5. **Simplify** the remaining integral

📊 DEFINITE INTEGRALS

Definition: $\int$[from a to b] f(x) dx = lim(n→∞) Σ[i=1 to n] f(xi)Δx

Fundamental Theorem of Calculus: $\int$[from a to b] f(x) dx = F(b) - F(a)

Where F(x) is any antiderivative of f(x).

Properties of Definite Integrals:

- **$\int$[from a to a] f(x) dx = 0**
- **$\int$[from a to b] f(x) dx = -$\int$[from b to a] f(x) dx**
- **$\int$[from a to b] f(x) dx + $\int$[from b to c] f(x) dx = $\int$[from a to c] f(x) dx**

🎯 APPLICATIONS OF INTEGRATION

Geometric Applications:

- **Area under curves:** $\int$[from a to b] f(x) dx
- **Area between curves:** $\int$[from a to b] |f(x) - g(x)| dx
- **Volume of solids:** Using disk/washer/shell methods
- **Arc length:** $\int$[from a to b] $\sqrt{(1 + (dy/dx)^2)}$ dx

Physical Applications:

- **Work done:** W = $\int$ F(x) dx
- **Center of mass:** Using moment calculations
- **Fluid pressure:** Against surfaces
- **Electric field:** From charge distributions

Economic Applications:

- **Consumer surplus:** Area between demand curve and price
- **Producer surplus:** Area between price and supply curve
- **Total cost:** From marginal cost function
- **Present value:** Of continuous income streams